

# *Bioinformatics of Proteins*

- Atomic Properties
- The Folding Problem
- Structure Alignments
- Structure Prediction

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# *Proteins in Bioinformatics*



- How do we represent structures for computation?
- How do we compare structures *in silico*?
- How do we classify structures hierarchically?

## *The Plan*



- Apply constraints of chemistry
  - Bond Lengths, Bond Angles, Dihedral (Torsion) Angles
- Place in Coordinate Frame
  - Cartesian, Internal, & Object Based Frames
- Compare Structures with  $i$  discrete components
  - Root Mean Squared Deviation

# *Basic Measurements*



- Bond Lengths
- Bond Angles
- Dihedral (Torsion) Angles

# *Bond Length*



- Bond Length fixed, given any scenario
- Depends on **type** of bond: single, double, triple, hybridization too
- Depends on which two atoms
- C-H is 1.0 Angstroms, C-C is 1.5 Angstroms
- Bond Length is a function of Spatial Position of the two atoms

# *Bond Length is Euclidean Distance*

For  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ ,  
$$d = \{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{1/2}$$

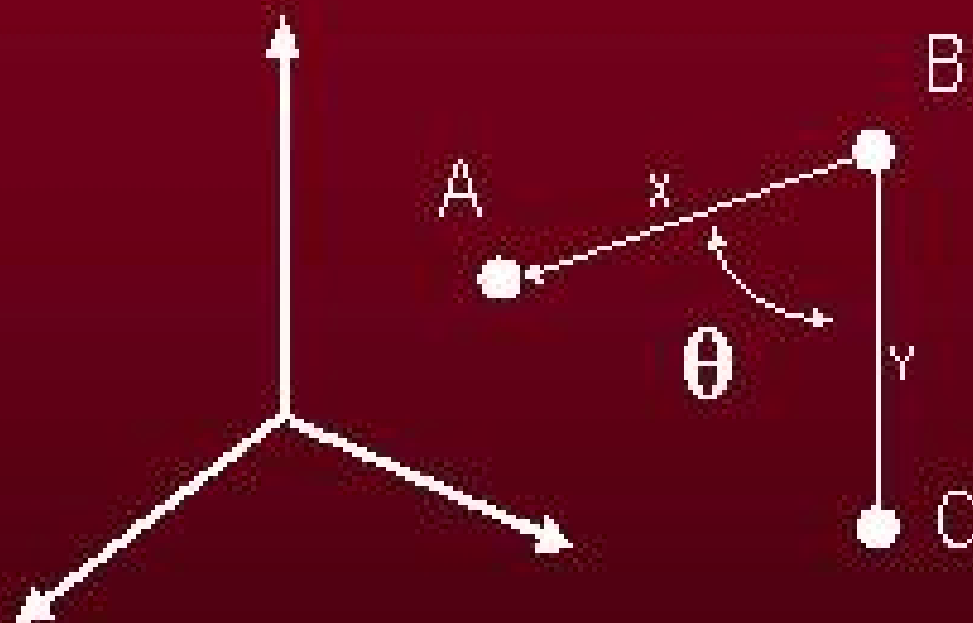
- Some non-covalent distances are also constant in a peptide's backbone
- $C_{\alpha} - C_{\alpha}$  distance for consecutive amino acids is constant too because of dihedral constraints

## *Bond Angles*



- Chemistry also fixes Bond Angles
- Depends on types of atoms, hybridization states, and number of lone electron pairs
- Range is 100 degrees to 180 degrees
- Bond Angles is a function of the spatial position of three atoms

# Computing Bond Angle



$$X \cdot Y = |X||Y|\cos(\theta)$$

$$\theta = \arccos(X \cdot Y / |X||Y|)$$

Angle can be computed by computing the arccosine of the dot product between unit vectors BA and BC.

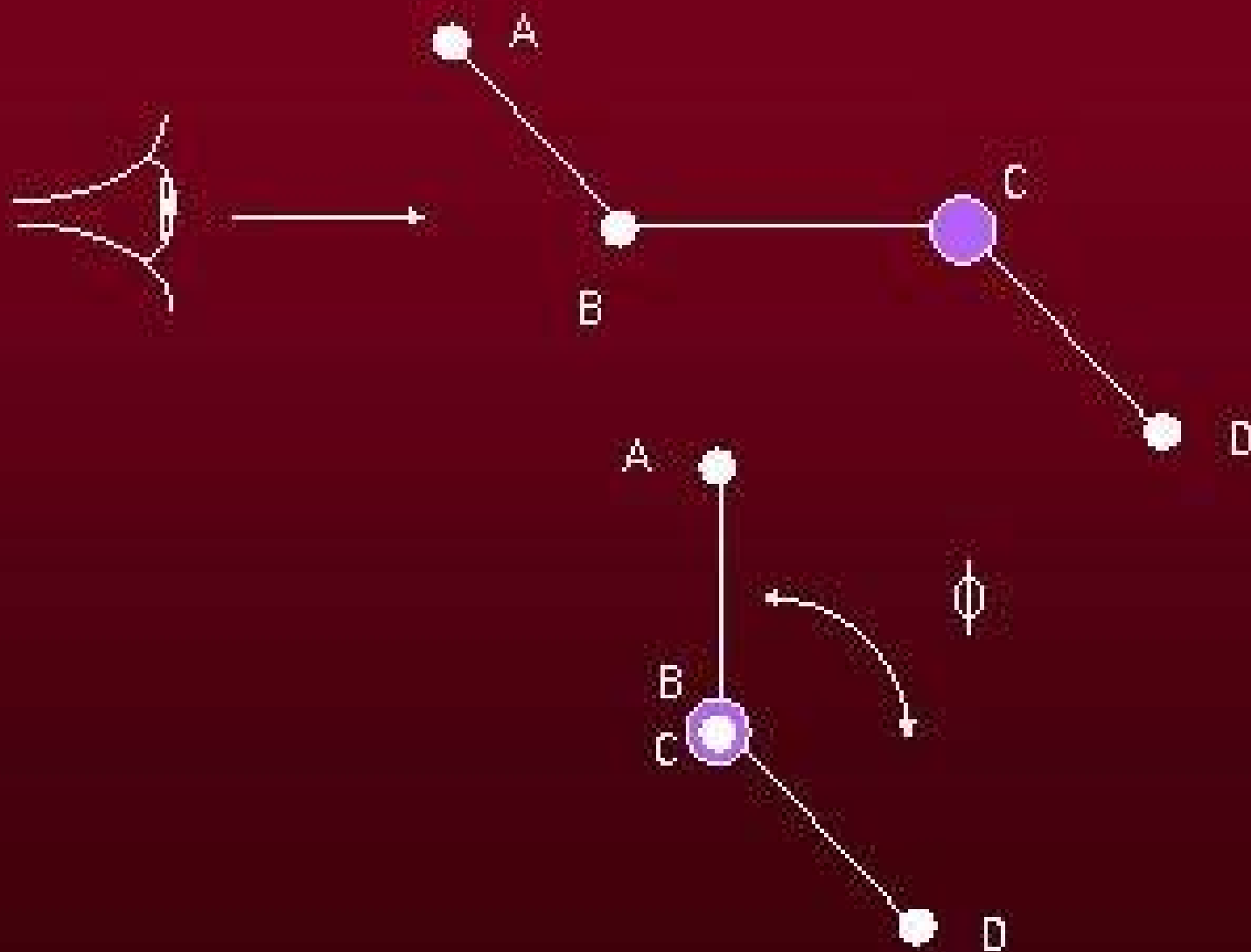


# *Dihedral Angles*



- These vary
- Range from 0 to 360 in principle
- Common in proteins are  $\phi$ ,  $\psi$ ,  $\omega$ , &  $\chi$
- Dihedral Angles are a function of the spatial position of four atoms in space

# Dihedral Angle



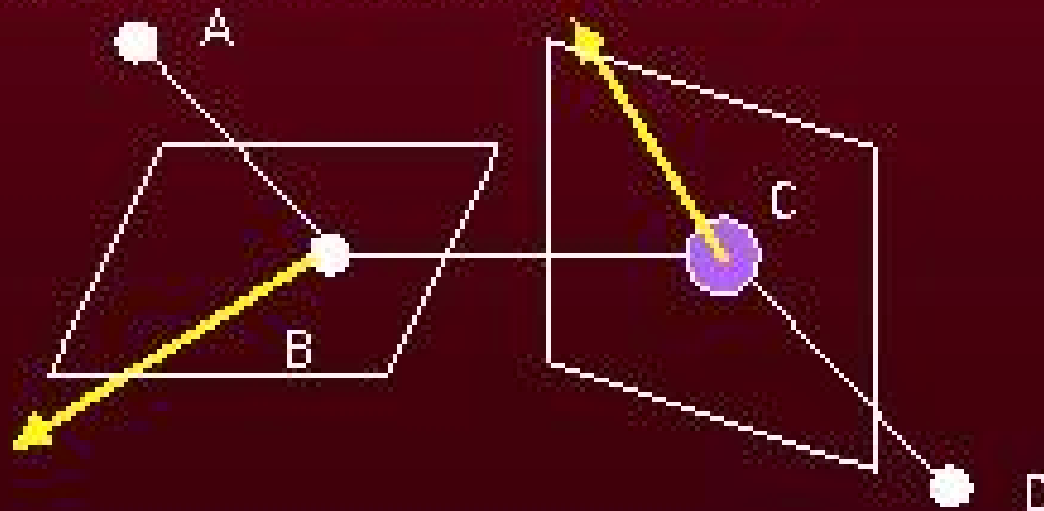
# Computing Dihedral Angle

Compute cross-product of BA and CB

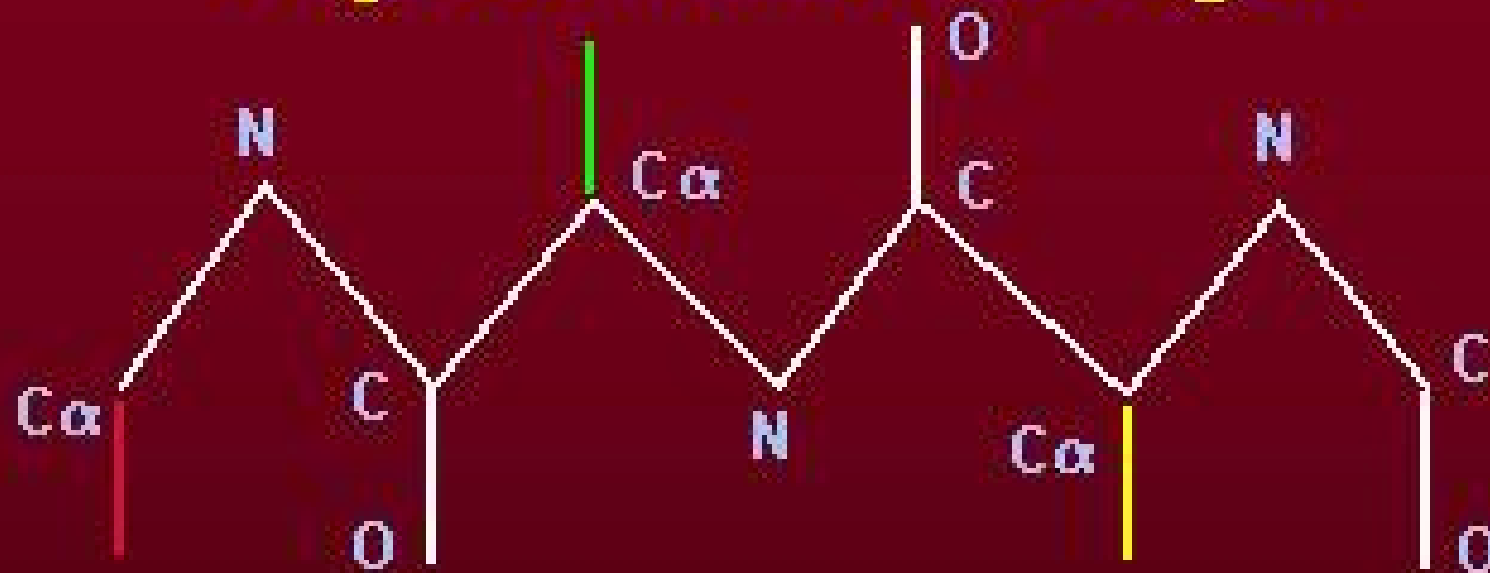
Compute cross-product of CB and DC

This produces two vectors perpendicular to the ABC plane and BCD plane.

Angle between those vectors (ala bond angle) is dihedral angle.  
Need to check if it is positive or negative.



# Important Dihedral Angles



$\omega$  (omega) = C $\alpha$  to C $\alpha$



$\psi$  (psi) = N to N

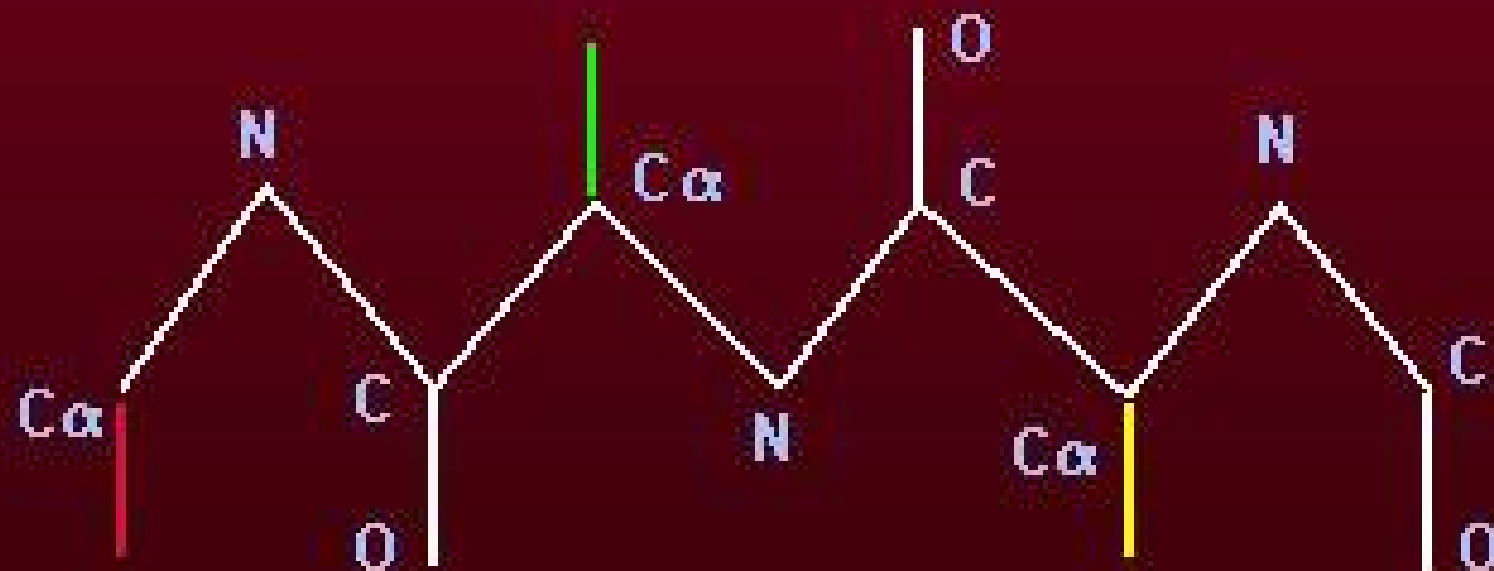


$\phi$  (phi) = C to C

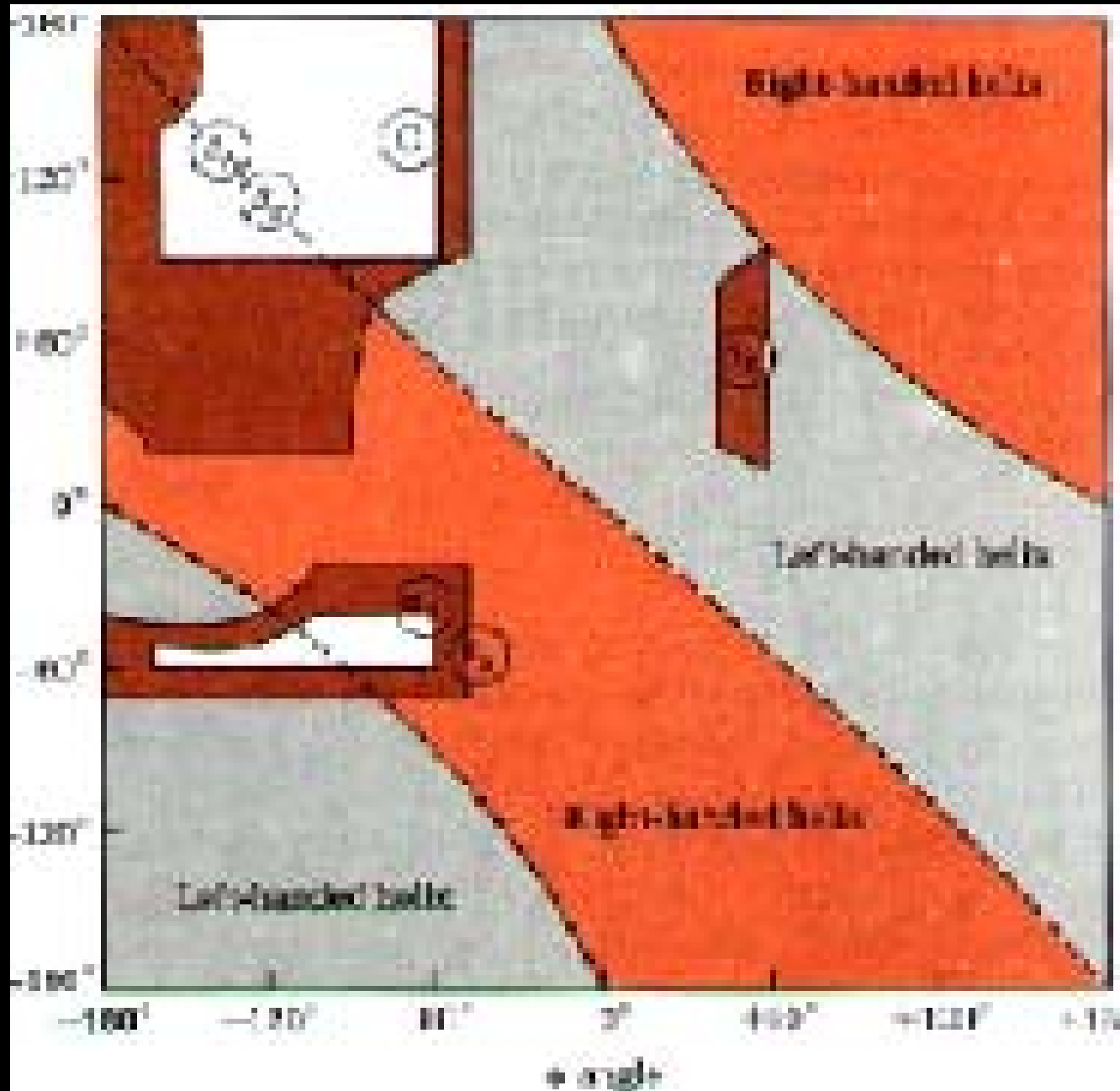


Omega is constant = 180 (C-N doesn't rotate)  
Phi, Psi have range of values (C $\alpha$ -N, N-C rotate)

The range is restricted by having things not bump into each other.



# Ramachandran Plot



Steric constraints restrict possible set of dihedral angles

# *Typical Secondary Structures have known Dihedral Angles*



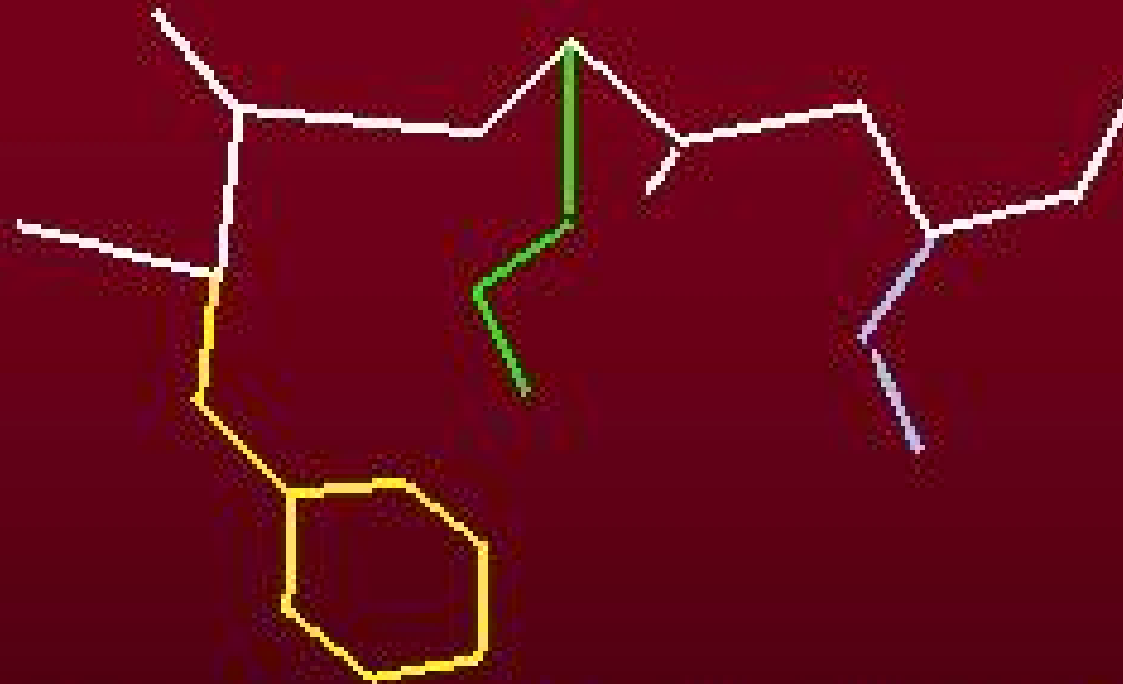
- Alpha Helix
  - $\Phi = -57$  degrees,  $\psi = -47$  degrees
- Parallel Beta Strand
  - $\Phi = -119$  degrees,  $\psi = 113$  degrees
- Antiparallel Beta Strand
  - $\Phi = -139$  degrees,  $\psi = 135$  degrees

# *Coordinate Frames*

- Cartesian Frame has orthonormal (x,y,z) basis & provides signed lengths for motion along each axis (used in Protein DataBase)
- But since bond lengths and angles are basically constant, why not just specify dihedral angles?
- Leads to **internal coordinate frame**



## Think about advantages



3 Peptide Units = 12 atoms = **36** coordinates OR **6** dihedral angles

3 Sidechains = 12 atoms = **36** coordinates OR **5** dihedral angles

**72** Cartesian Coordinates vs. **11** internal coordinates

# *Disadvantages of Internal Frame?*



- Basic computations (like Euclidean distance) are really difficult
- How about objects which aren't connected?
- Makes algorithms more complex sometimes

# *Object-Based Coordinate Frame*



- Certain part of proteins have less variability, like an alpha helix backbone
- Treat helix backbone as rigid object
- Reduces number of parameters specified

# *Comparing Structures*



- Compare structures A & B
- Need to know which atoms in A correspond to which in B
  - Get this from BLAST
- Need to know position of all atoms
  - Get this from PDB

# Comparing Structures



- How closely can two structures be superimposed?
- Need an *objective function* to measure this
- If exactly the same, measure = 0
- If divergent structures, measure is large

# The RMSD

RMSD = root mean squared deviation

$$RMSD = \sqrt{\frac{\sum_i d_i^2}{N}}$$

where  $N$  is the number of atoms

$d_i$  is the distance between two atoms with index  $i$  from the two structures

**We want the minimum RMSD**



# *RMSD Algorithms*



- Greedy search around center of mass for lowest RMSD
  - Superimpose centers of mass
  - Calculate RMSD
  - Rotate slightly
  - Re-calculate RMSD, and chose lowest
- \*Method based on translation and rotation matrices\*
  - Algorithm based on eigenvectors



# *Advantages of RMSD*



- Nice behavior
  - 0 when identical, falls off continuously
- Easy to compute
- Units are natural (Angstroms)
- Commonly Used
- Similar structures show 1-3 Angstroms RMSD

## *Disadvantages of RMSD*



- All atoms are equally weighed
- Upper bound variable
- Significance cutoff increases as size increases

# *Case Study: Myoglobin Superfamily*



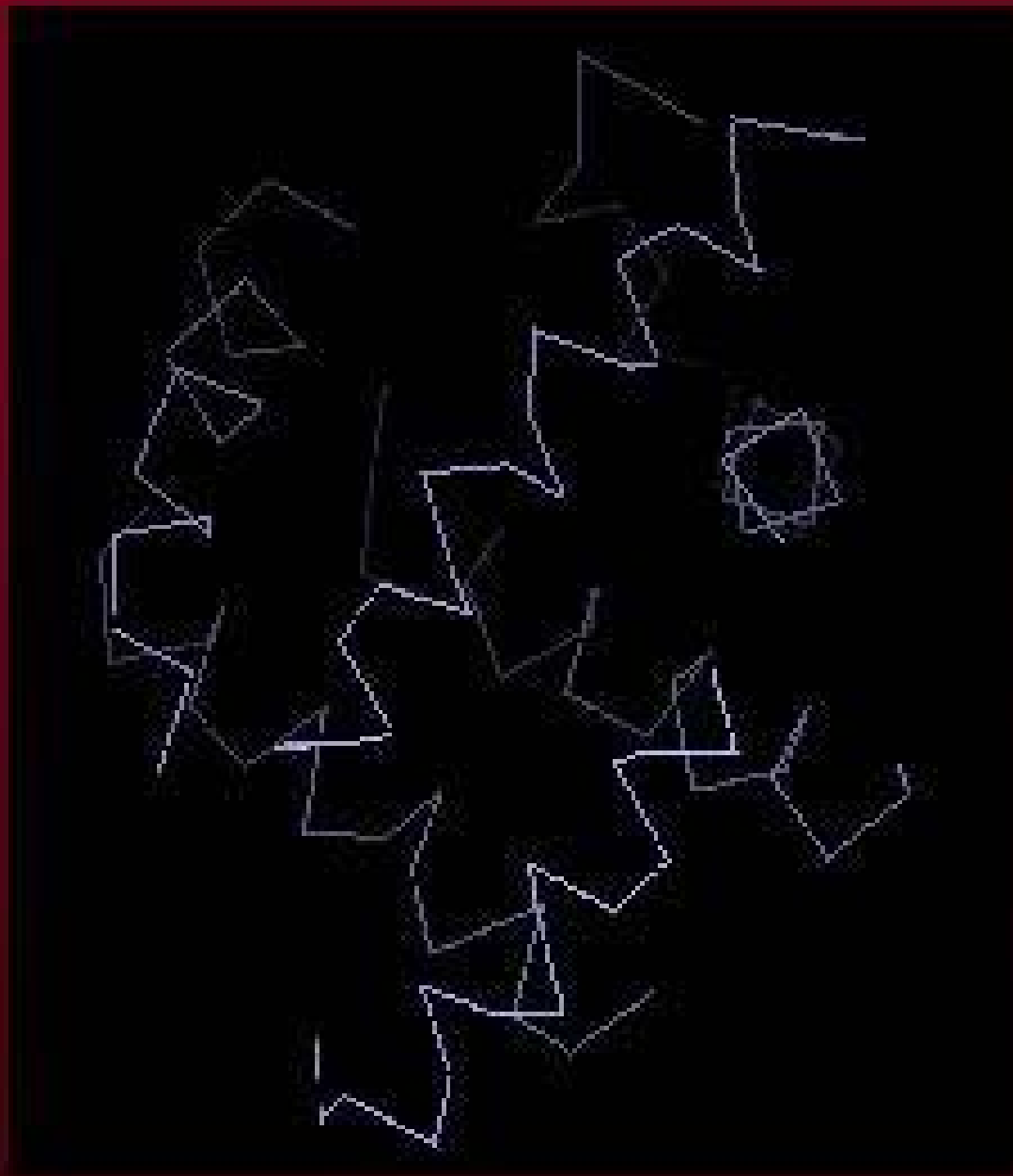
- Eight structures involved:
  - Sperm whale myoglobin
  - Sea hare myoglobin
  - Plant leghemoglobin
  - Sea lamprey hemoglobin
  - Human alpha & beta hemoglobin chains
  - Chironomous hemoglobin
  - Bloodworm hemoglobin
- Aligned by hand b/c of low a.a. identity
- 115 common positions

[illegible]

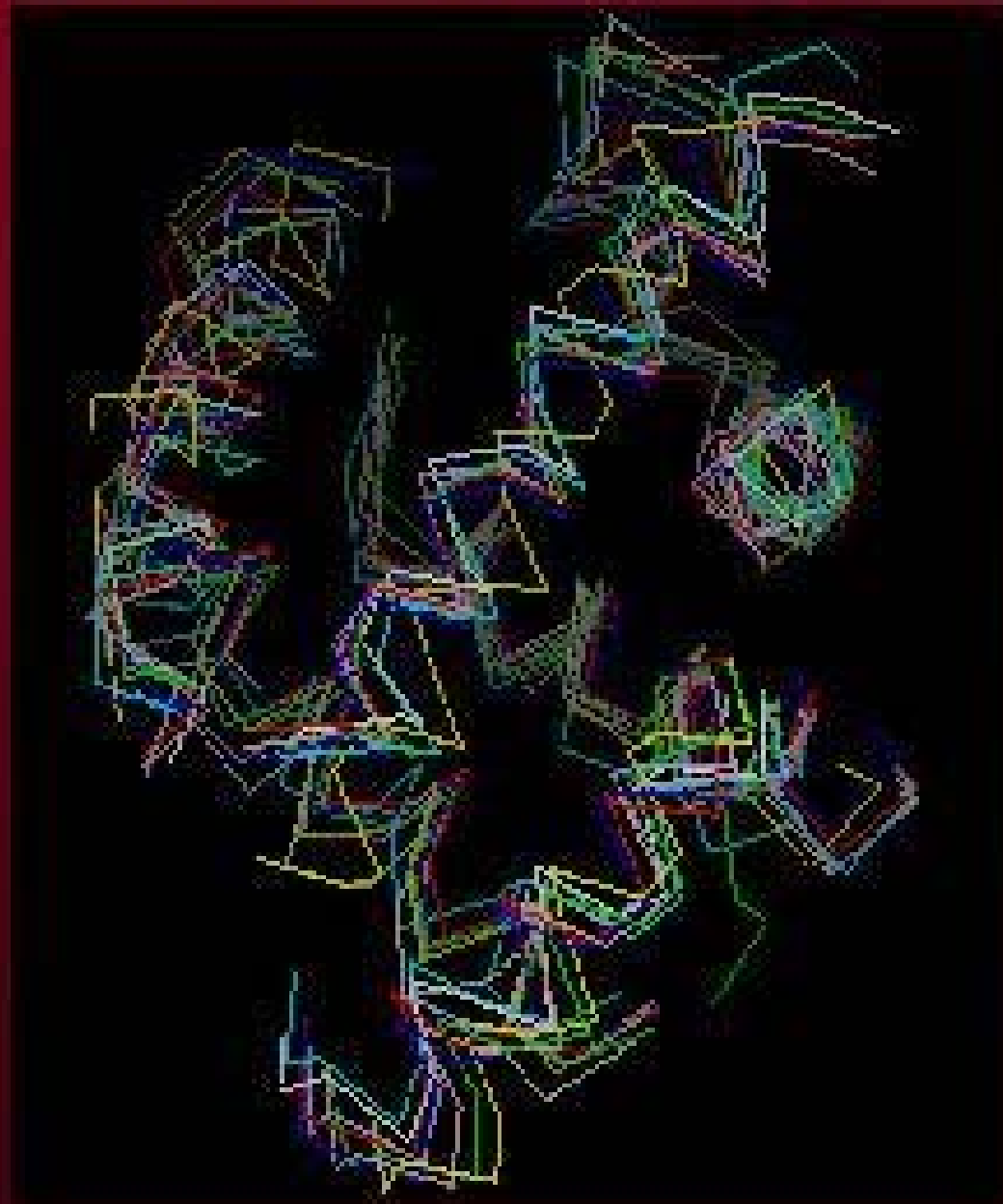
## *RMS for alpha carbons*



- $N(N-1)/2$  pairwise RMSs computed ( $N=8$ )
- Ranged from 1.22 to 3.16 Angstroms
- Average was 2.19 Angstroms



# Superimpose all on an average



## *Conclusions*



- Compute bond length, bond angles, dihedral angles
- Work in different coordinate frames
- Use RMSD for structure comparison
- Graphical superimposition can elucidate structural similarities & differences



# *The Protein Folding Problem*



- The Search Space
- Definitions of Energy
- Computing Free Energy
- The Energy Function
- MonteCarlo Methods
- Molecular Dynamics

# *The Folding Problem*



- How does the linear a.a. sequence fold to the 3-D shape off the ribosome?
- And more broadly, how do we get the 3-D structure given a linear a.a. sequence?

# *The Input Space*



- Linear amino acid sequence
- Structure of each amino acid and peptide backbone
  - Lists of atoms, bond lengths, bond angles
  - Ramachandran constraints on dihedral angles
- The media
  - Water and dissolved solutes (salts)

# *The Output Space*



- The 3-D coordinates of the protein in some frame
- Partial Answers:
  - 3-D structure of active site
  - Location in linear sequence of secondary structure
  - Prediction of “*class*” or “*family*” of the protein

## *Why should we care?*



- Sequence ---> Structure ---> Function
- Structure very useful for Drug Design
- Hard to get structures experimentally
  - X-ray crystallography (80%) 1-2 Å
  - Nuclear Magnetic Resonance (20%) 1-3 Å
  - Cryo Electron Microscopy (<<1%) 7-10 Å

# *How hard is the problem?*



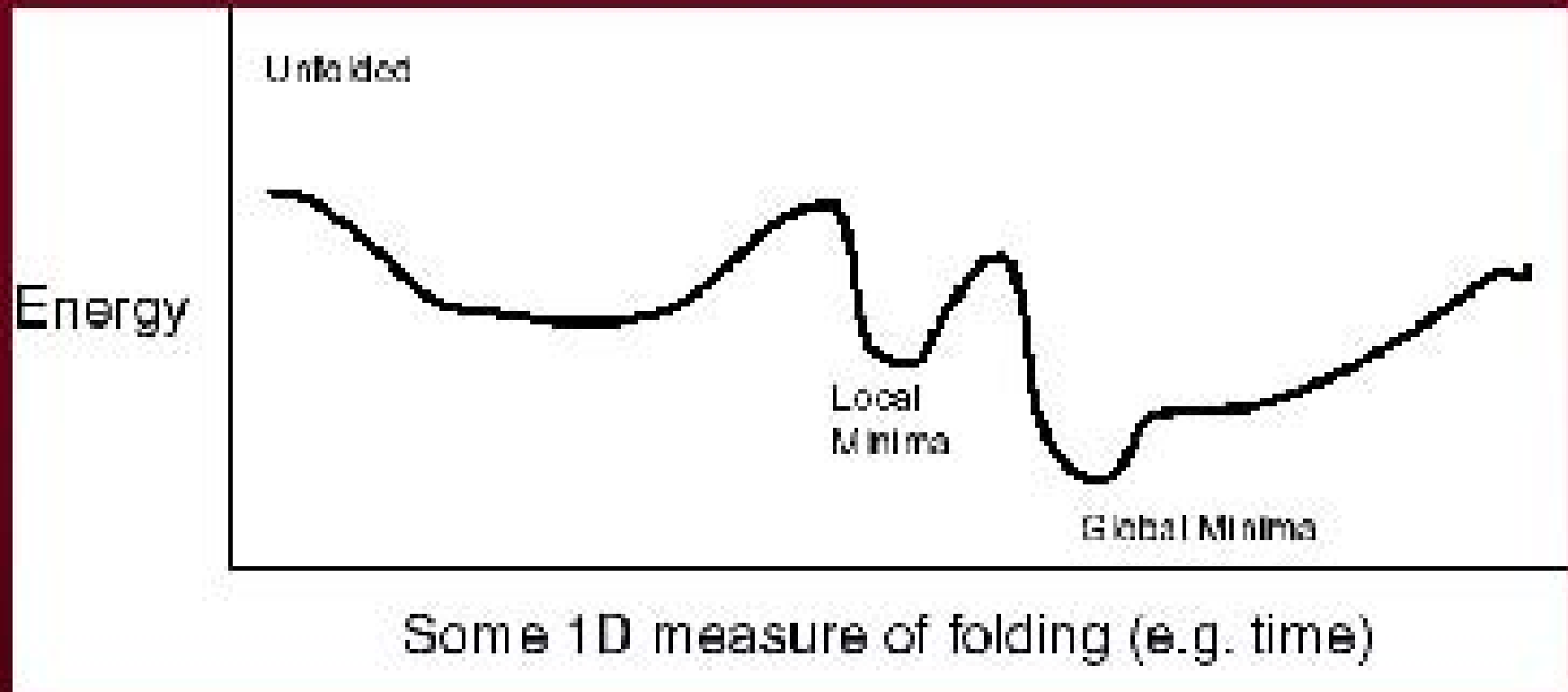
## Very Hard

- Huge search space
- For a 100 a.a. chain, assume each a.a. can be in either alpha, beta, or coil state (simplification)
- $3^{100} = 5 * 10^{47}$  possible distinct folds
- At 1 fold every 0.10 ps, it takes  $10^{27}$  years
- Universe is  $10^{10}$  years old

## *Why is the problem hard?*



- How do we know when we have the “correct” fold?
- Need to measure interactions between a.a.’s, water, and other molecules
- You are folding proteins right now
- You do it in seconds





# *Sampling the Output Space*



- Secondary structure occurs regularly
  - Can form locally, independent of global structure
- Steric constraints eliminate some possibilities
- Maybe a nonrandom search?
  - Local structure can form and induce cascades

# *Gibbs Free Energy*



- $\Delta G = \Delta H - T\Delta S$
- Free Energy = Enthalpic Energy - Entropic Energy  
 $\Delta H$  = benefits of interactions (negative for folding)  
 $T\Delta S$  = costs of imposing order (negative for folding)
- Proteins fold **because**  $\Delta H < T\Delta S$
- Usually just by a narrow margin

# *Entropy*



- High entropy means disorder
- $S = k \ln \Omega$ , where  $\Omega = \#$  arrangements
- If only 1 state is allowed  $\Omega = 1$ , and  $S=0$
- Often hard to compute by statistical mechanics
- Turn to a more classical approach



# *Energy*

- Total Energy = Potential + Kinetic
- $E = U + K$
- Use Newtonian physical approximations
  - Atoms and bonds as balls and springs
- Seek energy minima

# *Writing an Energy Function*



- Bond Lengths
- Bond Angles
- Dihedral Angles (Ramachandran constraints)
- Packing term (nature abhors a vacuum)
- Electrostatic interactions

# Potential Energy function:

$$\begin{aligned} U = & \sum_{bonds} K_b (b_i - b_o)^2 \\ & + \sum_{angles} K_\theta (\theta_i - \theta_o)^2 \\ & + \sum_{dihedrals} K_\phi [1 - \cos(n\phi_i + \delta)] \\ & + \sum_{pairs} \epsilon \left[ \left( \frac{r_o}{r_{ij}} \right)^{12} - 2 \left( \frac{r_o}{r_{ij}} \right)^6 \right] \\ & + \sum_{charges} \frac{q_i q_j}{r_{ij}} \end{aligned}$$

The potential energy is a function of all atomic coordinates.

The sum of all these terms creates a function with many local minima and maxima.

Very hard to sample this function well.

$$\begin{aligned} U = & \sum_{bonds} K_b (b_i - b_o)^2 \\ & + \sum_{angles} K_\theta (\theta_i - \theta_o)^2 \\ & + \sum_{dihedrals} K_\phi [1 - \cos(n\phi_i + \delta)] \\ & + \sum_{pairs} \epsilon \left[ \left( \frac{r_o}{r_{ij}} \right)^{12} - 2 \left( \frac{r_o}{r_{ij}} \right)^6 \right] \\ & + \sum_{charges} \frac{q_i q_j}{r_{ij}} \end{aligned}$$

# DALI optimizes with a MonteCarlo (Metropolis, 1953) algorithm.

Basic idea: iterative improvement by random walk through search space, with occasional excursions into “non-optimal” territory.

Only by allowing non-optimal jumps, do you allow yourself to leave local optima.





# *MonteCarlo Algorithm*



- Choose a starting position  $P$
- Evaluate the objective scoring function  $S$
- Perturb the current position (randomly or otherwise) to  $P'$  and compute  $S'$
- If  $S' < S$ , let  $P = P'$
- Else let  $P = P'$  with probability  $e^{\beta(S'-S)}$
- Loop

# *Relative Energies*



- Hydrogen Bond -5.0 kcal/mol
- Change in Bond Angle by 10 degrees +2.0 kcal/mol
- Stretch bond length by 0.1 Angstroms +2.5 kcal/mol
- Pack two atoms snugly -0.2 kcal/mol
- Break a bond +100 kcal/mol
- Bring two +1 charges to 3 Angstroms +100 kcal/mol

# *Searching for Global Energy Minima*



- Search for atomic coordinates that minimize  $U$
- Generally finds only local minima
- Can use MonteCarlo algorithms,
- Need good (nonrandom) starting structure
- Works well for relaxing perturbations of known structures
- No water, no solutes included

# *Molecular Dynamics*



- $F(x,y,z) = -\text{Grad}[U(x,y,z)]$
- $F = m a$
- Simulate atomic paths by small linear motions
- To make small motions, need small time step

## *Time Steps*



- Bond stretching 0.01 ps
- Angle bending 0.1 ps
- Rotating methyl group 1.0 ps
- Water tumbling 10 ps
- Protein tumbling in water 10,000 ps
- Chemical Reaction 1,000,000 ps

Need time step = 0.001 ps = 1.00 femtoseconds!!!

# *Goals of Molecular Dynamics*



- Learn how protein moves in water
- Learn response to perturbation
- Fold proteins *ab initio*
- Run microseconds of simulation

# *Incorporate Experimental Facts*



- The part off the ribosome first doesn't necessarily fold first
- Secondary structure forms rapidly, making problem easier

# IBM Blue Gene = Fold a protein

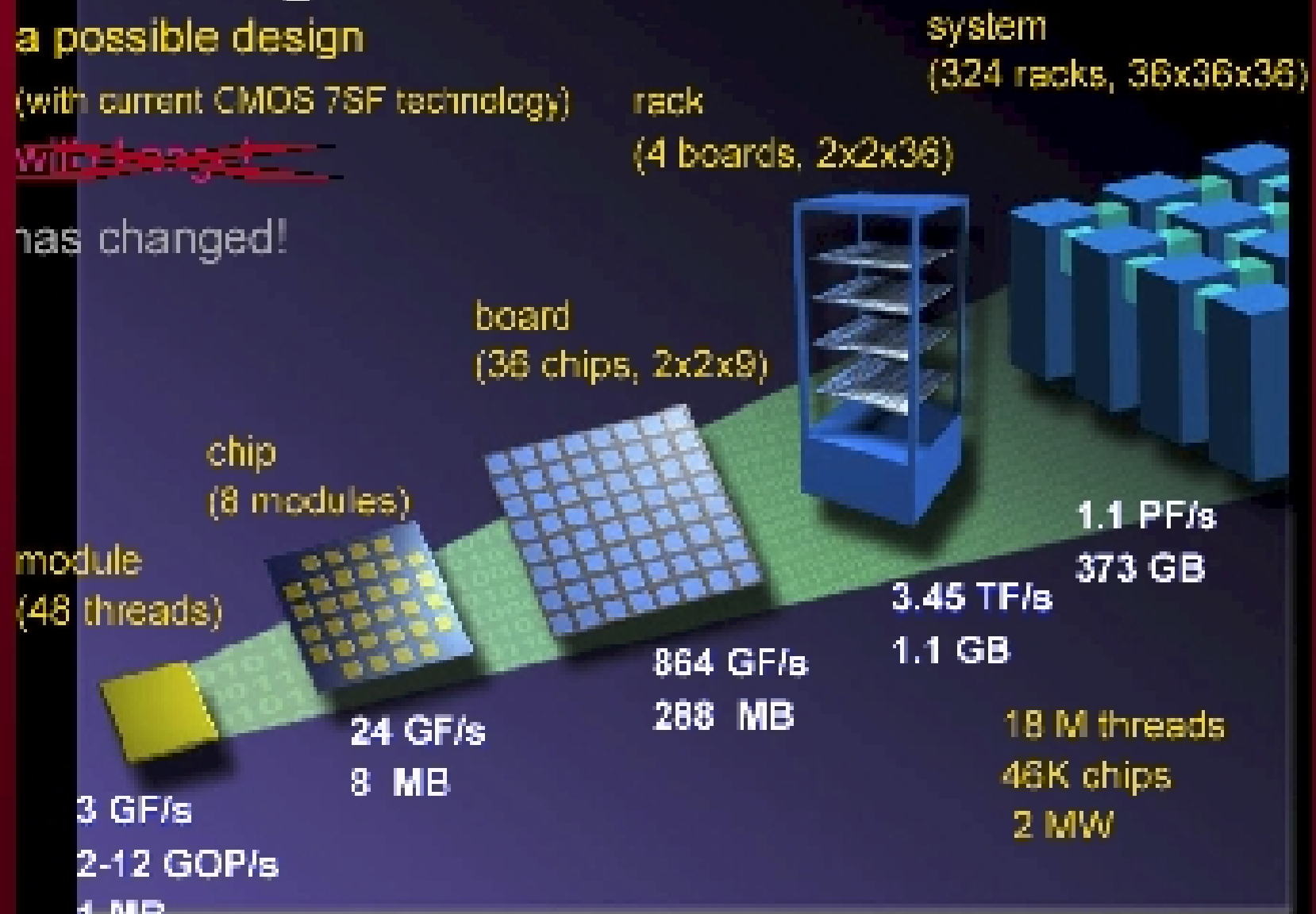
## Building Blue Gene

a possible design

(with current CMOS 7SF technology)

~~with budget~~

has changed!





# *Structure Alignment*



- Fit structure A with  $i$  elements to B with  $j$  elements
- Analogy

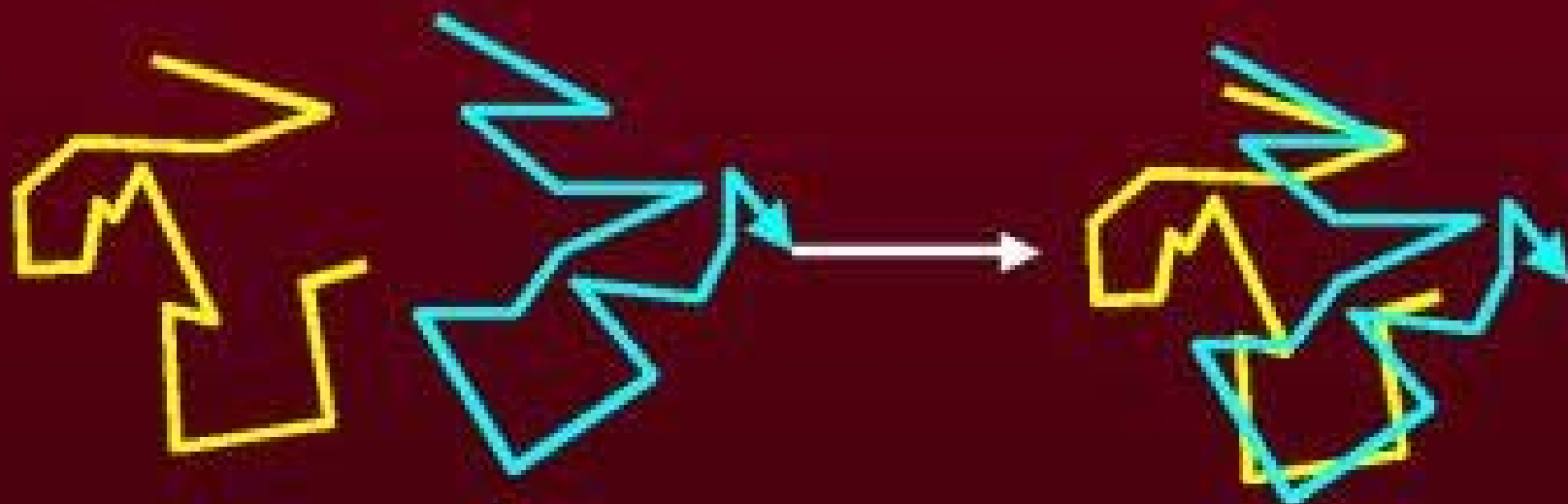
RMSD                       $i$  to  $i$     BLAST without gaps

Structure Alignment     $i$  to  $j$     BLAST with gaps

- Use RMS as tool in computing Structure Alignment

# Problem

Given a pair of molecular structures, find the correspondences between atoms that leads to the “best” alignment.



## Note:

“best” means the most atoms aligned with the lowest RMS.

Tradeoff: few atoms aligned very well vs. lots of atoms aligned not so well.



# *Criteria for Alignment*



- $i$  and  $j$
- % identity or similarity of aligned a.a.'s
- # of gaps
- Shared active site?

# *Why bother aligning?*



- As a check on sequence searches (BLAST)
- Make a hierarchy of classification of proteins
  - <http://scop.stanford.edu>
  - Alexei Murzin (manual) or Algorithmically
- Evaluate common ancestry

# Manual Clustering of Structures

(see Structural Classification of Proteins  
= SCOP at <http://scop.stanford.edu>)

- **Class**

- similar 2<sup>o</sup> structure
- all  $\alpha$ , all  $\beta$ ,  $\alpha + \beta$ ,  $\alpha/\beta$

- **Fold**

- major structural similarity
- similar arrangement of 2<sup>o</sup>

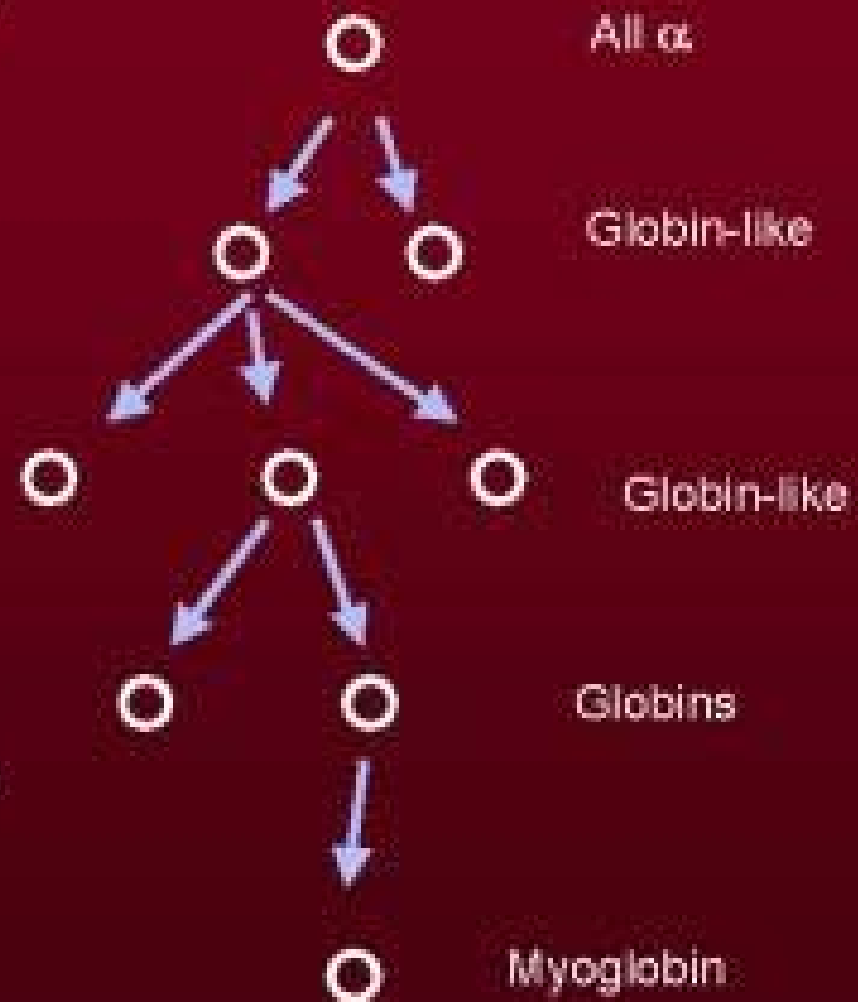
- **Superfamily** (topology)

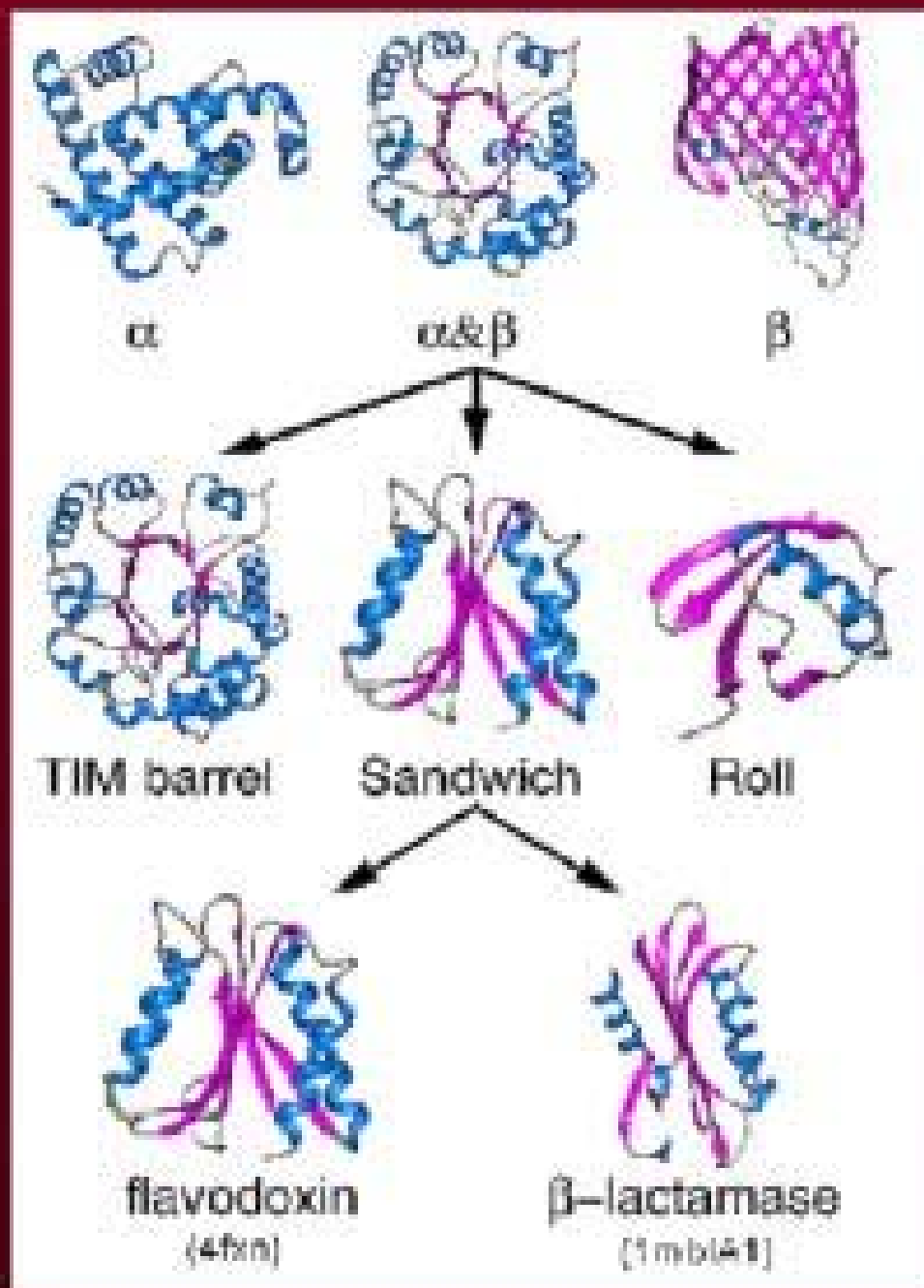
- probable common ancestry

- **Family**

- clear evolutionary relationship
- sequence similarity > 25%

- **Individual Protein**





# *Algorithms*



- STRUCTAL (Levitt, Subbiah, Gerstein)
- DALI (Holm, Sander)
- LOCK (Singh, Brutlag)



# *Folding vs.. Prediction*



- Folding gets to 3-D structure by simulating physical principles
  - Energy minimization
  - Molecular Dynamics
- Prediction gets to 3-D structure using statistical, theoretical, and/or empirical info
  - Just get structure, doesn't matter how

# *Asilomar Contest*



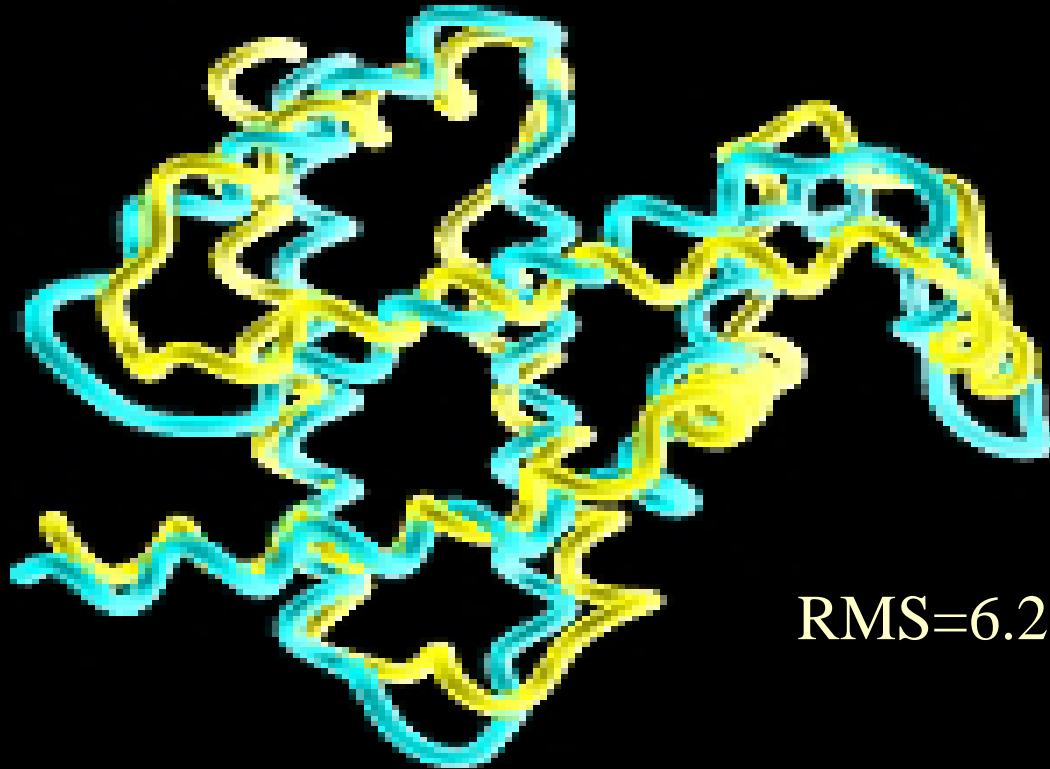
- Started 1994 and runs biannually
- Conference near Monterey
- “Meeting on Critical Assessment of Techniques for Protein Structure Prediction (CASP)”
  - Homology Modeling (>25% sequence identity)
  - Fold Recognition (20-25% sequence identity)
  - *Ab initio* prediction (no homology)

## *The Players*



- Experimentalists - gets structure empirically
- Predictors download sequence and minimal info
- Assessors use RMS, alignment to evaluate results of predictors algorithms

# *Evaluation*



RMS=6.2 Angstroms

# *Homology Modeling*



- Goal is final 3-D structure
- >70% homology works great
- PSI-BLAST helps a lot
- Energetic relaxation doesn't help without a good guess

# *Fold Recognition*



- Goal is to map regions of linear sequence to known folds in PDB
- Worked surprisingly well in 1994
  - Keeps getting a bit better
- Evaluate on RMS, electrostatics, hydrophobic burial, H-bonds, energetics
- Every Predictor got at least one right

# *Ab initio Prediction*



- Goal is secondary and/or 3-D structure
- Secondary
  - 66-77% correct
  - Errors not tolerable, need better techniques
- 3-D Structure
  - Rosetta Method

# *Rosetta Method*



- Break target into 9 a.a. stretches
- Search PDB for that stretch of 9
- Align 9 to best match in PDB
- Steal structure around 9 from PDB
- Shift frame by 1 in linear sequence
- Loop
- Create thousands of structures and average

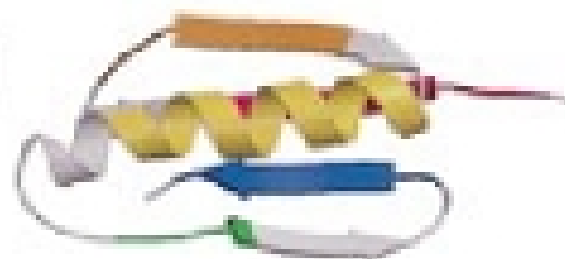


NATIVE



2ptl

PREDICTED



1sro



# Performance of Rosetta Method

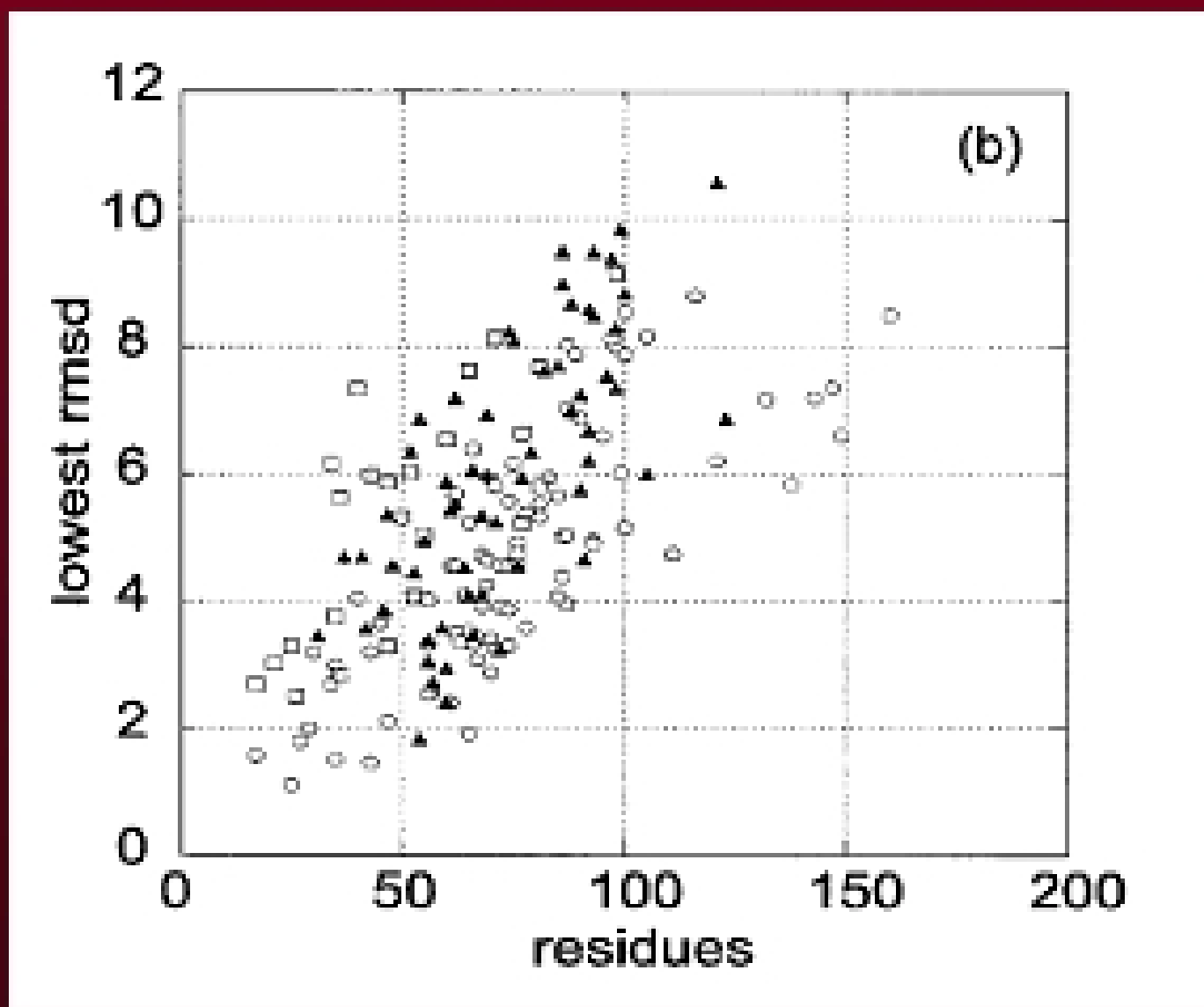


TABLE III. Fold Prediction Results\*

	Onaka	London	Saltburg	NIH	Scrpps	EMBL	Baltimore	Cambridge	Oxford
stanfex3	5 3a 3f (1.8)	>10	8 2a 2f (2.1)	9 1a 3f (2.1)	1 4a 3f (2.5)			1 2a 2f (2.7)	
penub	>10		1 2a 3f (2.9)	2 3a 4f (2.4)					
kon B	1 6f (2.2)	1 4f (2.0)						2 3f (2.7)	1 5f (2.7)
ntp	>10	>10	1 3a 2f (1.8)	10 3a 2f (1.6)					
synapto	6 7f (2.4)	1 7f (2.4)	>10		5 7f (2.3)		1 6f (2.2)	4 6f (1.9)	>10
ppdk 3	3 3a 3f (2.4)	6 2a 3f (2.8)		1 3a 4f (1.0)	4 2a 3f (2.8)				
pcna			>10						
xylose			1 7a 8f (2.8)					1 6a 7f (2.9)	
ppdk 4	1 8a 8f (2.5)	1 8a 8f (2.4)		8 8a 7f (2.8)	3 8a 7f (2.8)	>10			
phlg	3 5a 8f (2.7)	1 5a 7f (2.7)	10 7a 8f (2.9)	2 8a 7f (2.6)	>10				
kon A	4 5a 7f (3.0)	1 4a 6f (2.5)							
myosin	2 3a 7f (2.7)		5 7a 7f (2.7)	10 7a 7f (2.3)	4 7a 8f (2.8)				1 8a 8f (2.6)

Predictors are along top row. Target sequences along first column. Dark grey means bad prediction, light gray pretty good, white very good. Hatched means no prediction. Upper left corner shows rank of best answer among list submitted by predictors (also shows fold used to make prediction, shift error and general protein class)

# *Acknowledgments*



- Doug Brutlag
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